New Computational Methods for Compression and Recovery of Random Vectors

Coloquio de Matemática Aplicada

Escuela de Matemáticas
Instituto Tecnológico de Costa Rica

Juan Pablo Soto Quirós
jusoto@itcr.ac.cr

9-04-2018

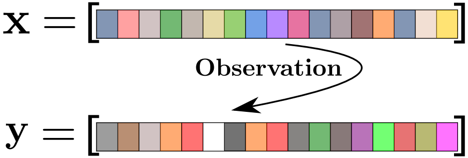
Consider the random vectors

- $\mathbf{x}$ (*source vector*)

$\mathbf{x} =$ 

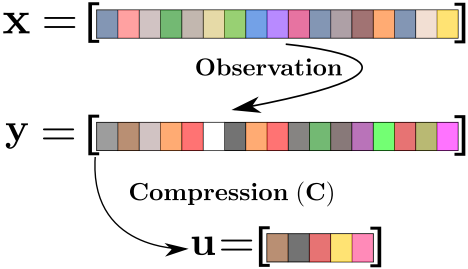
Consider the random vectors

- $\mathbf{x}$ (*source vector*)
- $\mathbf{y}$ (*observed vector*)



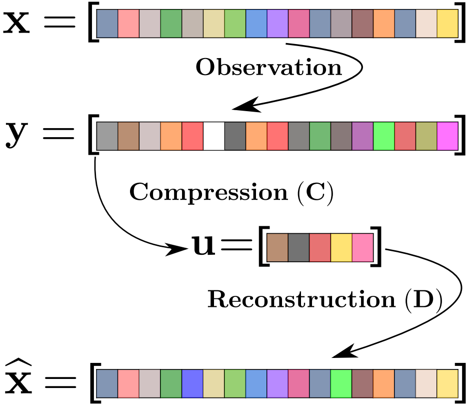
Consider the random vectors

- $\mathbf{x}$ (*source vector*)
- $\mathbf{y}$ (*observed vector*)
- $\mathbf{u}$ (*compressed vector*)

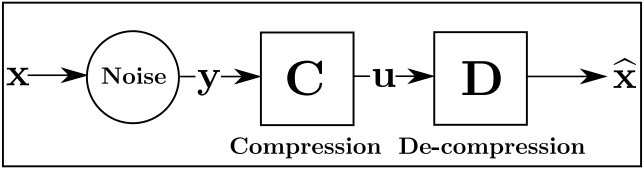


Consider the random vectors

- $\mathbf{x}$ (*source vector*)
- $\mathbf{y}$ (*observed vector*)
- $\mathbf{u}$ (*compressed vector*)
- $\widehat{\mathbf{x}}$ (*reconstructed vector*)



A Block Diagram Representation



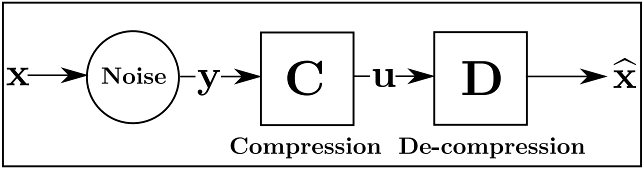
Statement of the Problem

Find optimal **new** models $\mathbf{C}$ and $\mathbf{D}$ such that

$$\hat{\mathbf{x}} \approx \mathbf{x}$$

and **improve the accuracy of known methods.**

A Block Diagram Representation



Statement of the Problem

Find optimal **new** models $\mathbf{C}$ and $\mathbf{D}$ such that

$$\hat{\mathbf{x}} \approx \mathbf{x}$$

and **improve the accuracy of known methods.**

1. Statement of the Problem
2. Literature Review
3. Our Contribution
4. Conclusions and Future Work

Special Notation

- $\mathbf{x} \in L^2(\Omega, \mathbb{R}^m)$, $\mathbf{y} \in L^2(\Omega, \mathbb{R}^n)$, $\mathbf{u} \in L^2(\Omega, \mathbb{R}^r)$ and $\hat{\mathbf{x}} \in L^2(\Omega, \mathbb{R}^m)$ random vectors.
- r is the dimension of compressed vector $\mathbf{u}$, and $r \leqslant \min\{m, n\}$.
- E_{xy} represents the covariance matrix of $\mathbf{x}$ and $\mathbf{y}$.
- $A^\dagger$ denotes the pseudo-inverse of A.
- $B^{1/2}$ is a square root of B, such that $B = B^{1/2}B^{1/2}$.

Special Notation

- $\mathbf{x} \in L^2(\Omega, \mathbb{R}^m)$, $\mathbf{y} \in L^2(\Omega, \mathbb{R}^n)$, $\mathbf{u} \in L^2(\Omega, \mathbb{R}^r)$ and $\hat{\mathbf{x}} \in L^2(\Omega, \mathbb{R}^m)$ random vectors.
- r is the dimension of compressed vector $\mathbf{u}$, and $r \leqslant \min\{m, n\}$.
- E_{xy} represents the covariance matrix of $\mathbf{x}$ and $\mathbf{y}$.
- $A^\dagger$ denotes the pseudo-inverse of A.
- $B^{1/2}$ is a square root of B, such that $B = B^{1/2}B^{1/2}$.

Special Notation

- $\mathbf{x} \in L^2(\Omega, \mathbb{R}^m)$, $\mathbf{y} \in L^2(\Omega, \mathbb{R}^n)$, $\mathbf{u} \in L^2(\Omega, \mathbb{R}^r)$ and $\hat{\mathbf{x}} \in L^2(\Omega, \mathbb{R}^m)$ random vectors.
- r is the dimension of compressed vector $\mathbf{u}$, and $r \leqslant \min\{m, n\}$.
- E_{xy} represents the covariance matrix of $\mathbf{x}$ and $\mathbf{y}$.
- $A^\dagger$ denotes the pseudo-inverse of A.
- $B^{1/2}$ is a square root of B, such that $B = B^{1/2}B^{1/2}$.

Special Notation

- $\mathbf{x} \in L^2(\Omega, \mathbb{R}^m)$, $\mathbf{y} \in L^2(\Omega, \mathbb{R}^n)$, $\mathbf{u} \in L^2(\Omega, \mathbb{R}^r)$ and $\widehat{\mathbf{x}} \in L^2(\Omega, \mathbb{R}^m)$ random vectors.
- r is the dimension of compressed vector $\mathbf{u}$, and $r \leqslant \min\{m, n\}$.
- E_{xy} represents the covariance matrix of $\mathbf{x}$ and $\mathbf{y}$.
- $A^\dagger$ denotes the pseudo-inverse of A.
- $B^{1/2}$ is a square root of B, such that $B = B^{1/2}B^{1/2}$.

Special Notation

- $\mathbf{x} \in L^2(\Omega, \mathbb{R}^m)$, $\mathbf{y} \in L^2(\Omega, \mathbb{R}^n)$, $\mathbf{u} \in L^2(\Omega, \mathbb{R}^r)$ and $\widehat{\mathbf{x}} \in L^2(\Omega, \mathbb{R}^m)$ random vectors.
- r is the dimension of compressed vector $\mathbf{u}$, and $r \leqslant \min\{m, n\}$.
- E_{xy} represents the covariance matrix of $\mathbf{x}$ and $\mathbf{y}$.
- $A^\dagger$ denotes the pseudo-inverse of A.
- $B^{1/2}$ is a square root of B, such that $B = B^{1/2}B^{1/2}$.

Transform proposed by Brillinger (BT) [Bri75]

Define the minimization problem

$$\min_{\substack{D \in \mathbb{R}^{m \times r} \\ C \in \mathbb{R}^{r \times n}}} \mathbb{E}[\|\mathbf{x} - DC\mathbf{y}\|_2^2], \tag{1}$$

A solution of (1) is the BT and is given by

$$D^* = U_r \quad \text{and} \quad C^* = U_r E_{xy} E_{yy}^{-1}.$$

- Columns of U_r are the eigenvectors of first r eigenvalues of $E_{xy} E_{yy}^{-1} E_{yx}$.

1. E_{yy} is nonsingular.
2. Solution is not unique ($DC = \underbrace{DP}\overbrace{P^{-1}C} = \tilde{D}\tilde{C}$)

Transform proposed by Brillinger (BT) [Bri75]

Define the minimization problem

$$\min_{\substack{D\in\mathbb{R}^{m\times r}\\ C\in\mathbb{R}^{r\times n}}} \mathbb{E}[\|\mathbf{x} - DC\mathbf{y}\|_2^2], \tag{1}$$

A solution of (1) is the BT and is given by

$$D^* = U_r \quad \text{and} \quad C^* = U_r E_{xy} E_{yy}^{-1}.$$

- Columns of U_r are the eigenvectors of first r eigenvalues of $E_{xy}E_{yy}^{-1}E_{yx}$.

1. E_{yy} is nonsingular.
2. Solution is not unique ($DC = \underbrace{DP}\overbrace{P^{-1}C} = \tilde{D}\tilde{C}$)

Transform proposed by Brillinger (BT) [Bri75]

Define the minimization problem

$$\min_{\substack{D \in \mathbb{R}^{m \times r} \\ C \in \mathbb{R}^{r \times n}}} \mathbb{E}[\|\mathbf{x} - DC\mathbf{y}\|_2^2], \qquad (1)$$

A solution of (1) is the BT and is given by

$$D^* = U_r \quad \text{and} \quad C^* = U_r E_{xy} E_{yy}^{-1}.$$

- Columns of U_r are the eigenvectors of first r eigenvalues of $E_{xy}E_{yy}^{-1}E_{yx}$.

(i) E_{yy} is nonsingular.

(ii) Solution is not unique ($DC = \underbrace{DP}\overbrace{P^{-1}C} = \tilde{D}\tilde{C}$)

Generalized Karhunen-Loeve Transform (GKLT) [HL98]

Define the minimization problem

$$\min_{\substack{F \in \mathbb{R}^{m \times n} \\ \text{rank}(F) \leqslant r}} \mathbb{E}[\|\mathbf{x} - F\mathbf{y}\|_2^2], \tag{2}$$

A solution of (2) is the GKLT and is given by

$$F^* = [E_{xy} E_{yy}^{1/2\dagger}]_r E_{yy}^{1/2\dagger}.$$

- $[\bullet]_r$ denotes a truncated SVD taken with first r single values.

(i) If rank$(F) \leqslant r$, then $F = BA$, for some $B \in \mathbb{R}^{m \times r}$ and $A \in \mathbb{R}^{r \times n}$.

(ii) Solution is not unique ([TH07]).

(iii) $F^* = D^* C^*$.

Generalized Karhunen-Loeve Transform (GKLT) [HL98]

Define the minimization problem

$$\min_{\substack{F \in \mathbb{R}^{m \times n} \\ \text{rank}(F) \leqslant r}} \mathbb{E}[\|\mathbf{x} - F\mathbf{y}\|_2^2], \tag{2}$$

A solution of (2) is the GKLT and is given by

$$F^* = [E_{xy} E_{yy}^{1/2\dagger}]_r E_{yy}^{1/2\dagger}.$$

- $[\bullet]_r$ denotes a truncated SVD taken with first r single values.

1. If rank$(F) \leqslant r$, then $F = BA$, for some $B \in \mathbb{R}^{m \times r}$ and $A \in \mathbb{R}^{r \times n}$.
2. Solution is not unique ([TH07]).
3. $F^* = D^* C^*$.

Generalized Karhunen-Loeve Transform (GKLT) [HL98]

Define the minimization problem

$$\min_{\substack{F \in \mathbb{R}^{m \times n} \\ \text{rank}(F) \leqslant r}} \mathbb{E}[\|\mathbf{x} - F\mathbf{y}\|_2^2], \tag{2}$$

A solution of (2) is the GKLT and is given by

$$F^* = [E_{xy} E_{yy}^{1/2\dagger}]_r E_{yy}^{1/2\dagger}.$$

- $[\bullet]_r$ denotes a truncated SVD taken with first r single values.

1. If rank$(F) \leqslant r$, then $F = BA$, for some $B \in \mathbb{R}^{m \times r}$ and $A \in \mathbb{R}^{r \times n}$.
2. Solution is not unique ([TH07]).
3. $F^* = D^* C^*$.

Generalized Karhunen-Loeve Transform (GKLT) [HL98]

Define the minimization problem

$$\min_{\substack{F \in \mathbb{R}^{m \times n} \\ \text{rank}(F) \leqslant r}} \mathbb{E}[\|\mathbf{x} - F\mathbf{y}\|_2^2], \tag{2}$$

A solution of (2) is the GKLT and is given by

$$F^* = [E_{xy} E_{yy}^{1/2\dagger}]_r E_{yy}^{1/2\dagger}.$$

- $[\bullet]_r$ denotes a truncated SVD taken with first r single values.

1. If $\text{rank}(F) \leqslant r$, then $F = BA$, for some $B \in \mathbb{R}^{m \times r}$ and $A \in \mathbb{R}^{r \times n}$.
2. Solution is not unique ([TH07]).
3. $F^* = D^* C^*$.

- The BT and GKLT can be represent as

$$\mathcal{F}_1(\mathbf{y}) = F\mathbf{y} = DC\mathbf{y}.$$

- We said $\mathcal{F}_1$ is a **transform of first degree**.

- We define a **transform of second degree** as

$$\mathcal{F}_2(\mathbf{y}) = F_1\mathbf{y} + F_2\mathbf{y}^2.$$

- The BT and GKLT can be represent as

$$\mathcal{F}_1(\mathbf{y}) = F\mathbf{y} = DC\mathbf{y}.$$

- We said $\mathcal{F}_1$ is a **transform of first degree**.

- We define a **transform of second degree** as

$$\mathcal{F}_2(\mathbf{y}) = F_1\mathbf{y} + F_2\mathbf{y}^2.$$

- The BT and GKLT can be represent as

$$\mathcal{F}_1(\mathbf{y}) = F\mathbf{y} = DC\mathbf{y}.$$

- We said $\mathcal{F}_1$ is a **transform of first degree**.

- We define a **transform of second degree** as

$$\mathcal{F}_2(\mathbf{y}) = F_1\mathbf{y} + F_2\mathbf{y}^2.$$

GKLT of Second Degree (GKLT2) [TH01]

Let's define $\mathcal{F}_2(\mathbf{y}) = F_1\mathbf{y} + F_2\mathbf{y}^2$ and the problem

$$\min_{\substack{F_1, F_2 \in \mathbb{R}^{m\times n} \\ \text{rank}([F_1\ F_2]) \leqslant r}} \mathbb{E}[\|\mathbf{x} - \mathcal{F}_2(\mathbf{y})\|_2^2], \tag{3}$$

A solution of (3) is the GKLT2 and is given by

$$[F_1^* \ F_2^*] = [E_{xz}E_{zz}^{1/2\dagger}]_r E_{zz}^{1/2\dagger},$$

where $\mathbf{z} = \begin{bmatrix} \mathbf{y} \\ \mathbf{y}^2 \end{bmatrix} \in L^2(\Omega, \mathbb{R}^{2n})$.

1. The solution is not unique.
2. Under some conditions, the accuracy of GKLT2 is better than BT and GKLT.
3. An exact E_{xy^2} is difficult to find.

GKLT of Second Degree (GKLT2) [TH01]

Let's define $\mathcal{F}_2(\mathbf{y}) = F_1\mathbf{y} + F_2\mathbf{y}^2$ and the problem

$$\min_{\substack{F_1, F_2 \in \mathbb{R}^{m\times n} \\ \text{rank}([F_1\ F_2]) \leqslant r}} \mathbb{E}[\|\mathbf{x} - \mathcal{F}_2(\mathbf{y})\|_2^2], \tag{3}$$

A solution of (3) is the GKLT2 and is given by

$$[F_1^*\ F_2^*] = [E_{xz}E_{zz}^{1/2\dagger}]_r E_{zz}^{1/2\dagger},$$

where $\mathbf{z} = \begin{bmatrix} \mathbf{y} \\ \mathbf{y}^2 \end{bmatrix} \in L^2(\Omega, \mathbb{R}^{2n})$.

1. The solution is not unique.
2. Under some conditions, the accuracy of GKLT2 is better than BT and GKLT.
3. An exact E_{xy^2} is difficult to find.

GKLT of Second Degree (GKLT2) [TH01]

Let's define $\mathcal{F}_2(\mathbf{y}) = F_1\mathbf{y} + F_2\mathbf{y}^2$ and the problem

$$\min_{\substack{F_1, F_2 \in \mathbb{R}^{m\times n} \\ \text{rank}([F_1\ F_2]) \leqslant r}} \mathbb{E}[\|\mathbf{x} - \mathcal{F}_2(\mathbf{y})\|_2^2], \tag{3}$$

A solution of (3) is the GKLT2 and is given by

$$[F_1^*\ F_2^*] = [E_{xz}E_{zz}^{1/2\dagger}]_r E_{zz}^{1/2\dagger},$$

where $\mathbf{z} = \begin{bmatrix}\mathbf{y}\\ \mathbf{y}^2\end{bmatrix} \in L^2(\Omega, \mathbb{R}^{2n})$.

1. The solution is not unique.
2. Under some conditions, the accuracy of GKLT2 is better than BT and GKLT.
3. An exact E_{xy^2} is difficult to find.

GKLT of Second Degree (GKLT2) [TH01]

Let's define $\mathcal{F}_2(\mathbf{y}) = F_1\mathbf{y} + F_2\mathbf{y}^2$ and the problem

$$\min_{\substack{F_1, F_2 \in \mathbb{R}^{m\times n} \\ \text{rank}([F_1\ F_2]) \leqslant r}} \quad \mathbb{E}[\|\mathbf{x} - \mathcal{F}_2(\mathbf{y})\|_2^2], \tag{3}$$

A solution of (3) is the GKLT2 and is given by

$$[F_1^*\ F_2^*] = [E_{xz}E_{zz}^{1/2\dagger}]_r E_{zz}^{1/2\dagger},$$

where $\mathbf{z} = \begin{bmatrix} \mathbf{y} \\ \mathbf{y}^2 \end{bmatrix} \in L^2(\Omega, \mathbb{R}^{2n})$.

1. The solution is not unique.
2. Under some conditions, the accuracy of GKLT2 is better than BT and GKLT.
3. An exact E_{xy^2} is difficult to find.

Consider a transform $\mathcal{T} : L^2(\Omega, \mathbb{R}^n) \rightarrow \mathcal{T} : L^2(\Omega, \mathbb{R}^m)$ and define the problem

$$\min_{\mathcal{T}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}(\mathbf{y})\|_2^2].$$

Our contribution

- Develop new transforms $\mathcal{T}(\mathbf{y}) = \widehat{\mathbf{x}}$ such that allow **compression**, **de-compression** and **filtering** of vector $\mathbf{y}$.
- $\widehat{\mathbf{x}} \approx \mathbf{x}$.
- The accuracy of $\mathcal{T}(\mathbf{y})$ will be better that BT, GKLT and GKLT2.

Consider a transform $\mathcal{T} : L^2(\Omega, \mathbb{R}^n) \to \mathcal{T} : L^2(\Omega, \mathbb{R}^m)$ and define the problem

$$\min_{\mathcal{T}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}(\mathbf{y})\|_2^2].$$

Our contribution

- Develop new transforms $\mathcal{T}(\mathbf{y}) = \hat{\mathbf{x}}$ such that allow **compression**, **de-compression** and **filtering** of vector $\mathbf{y}$.
- $\widehat{\mathbf{x}} \approx \mathbf{x}$.
- The accuracy of $\mathcal{T}(\mathbf{y})$ will be better that BT, GKLT and GKLT2.

Consider a transform $\mathcal{T} : L^2(\Omega, \mathbb{R}^n) \to \mathcal{T} : L^2(\Omega, \mathbb{R}^m)$ and define the problem

$$\min_{\mathcal{T}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}(\mathbf{y})\|_2^2].$$

Our contribution

- Develop new transforms $\mathcal{T}(\mathbf{y}) = \widehat{\mathbf{x}}$ such that allow **compression**, **de-compression** and **filtering** of vector $\mathbf{y}$.
- $\widehat{\mathbf{x}} \approx \mathbf{x}$.
- The accuracy of $\mathcal{T}(\mathbf{y})$ will be better that BT, GKLT and GKLT2.

Consider a transform $\mathcal{T} : L^2(\Omega, \mathbb{R}^n) \to \mathcal{T} : L^2(\Omega, \mathbb{R}^m)$ and define the problem

$$\min_{\mathcal{T}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}(\mathbf{y})\|_2^2].$$

Our contribution

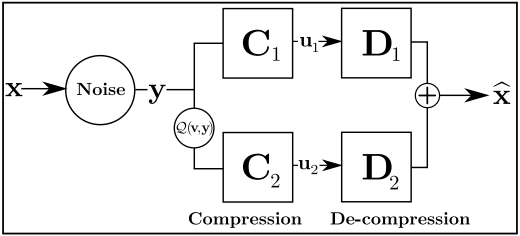
- Develop new transforms $\mathcal{T}(\mathbf{y}) = \widehat{\mathbf{x}}$ such that allow **compression**, **de-compression** and **filtering** of vector $\mathbf{y}$.
- $\widehat{\mathbf{x}} \approx \mathbf{x}$.
- The accuracy of $\mathcal{T}(\mathbf{y})$ will be better that BT, GKLT and GKLT2.

Transform Proposed 1 ($\mathcal{T}_1$)

Transform $\mathcal{T}_1$

$$\mathcal{T}_1(\mathbf{y}) = D_1C_1\mathbf{y} + D_2C_2\mathcal{Q}(\mathbf{v},\mathbf{y})$$

- $\mathcal{Q}(\mathbf{v},\mathbf{y}) = \mathbf{v} - E_{yv}E_{yy}^{\dagger}\mathbf{y}$.
- $\mathbf{v} \in L^2(\Omega, \mathbb{R}^q)$ is arbitrary



Problem with $\mathcal{T}_1$

Let's define $r_1 \leqslant \min\{m,n\}$ and $r_2 \leqslant \min\{m,q\}$. Solve

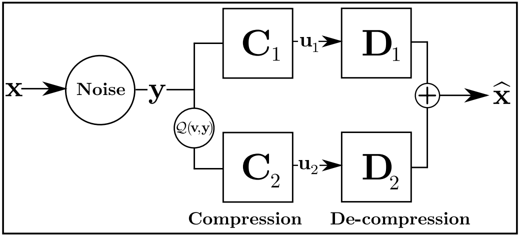
$$\min_{\substack{D_2\in\mathbb{R}^{m\times r_2}\\ C_2\in\mathbb{R}^{r_2\times q}}} \min_{\substack{D_1\in\mathbb{R}^{m\times r_1}\\ C_1\in\mathbb{R}^{r_1\times n}}} \mathbb{E}[\|\mathbf{x}-\mathcal{T}_1(\mathbf{y})\|_2^2].$$

Transform Proposed 1 ($\mathcal{T}_1$)

Transform $\mathcal{T}_1$

$$\mathcal{T}_1(\mathbf{y}) = D_1C_1\mathbf{y} + D_2C_2\mathcal{Q}(\mathbf{v},\mathbf{y})$$

- $\mathcal{Q}(\mathbf{v},\mathbf{y}) = \mathbf{v} - E_{yv}E_{yy}^{\dagger}\mathbf{y}$.
- $\mathbf{v} \in L^2(\Omega, \mathbb{R}^q)$ is arbitrary



Problem with $\mathcal{T}_1$

Let's define $r_1 \leqslant \min\{m,n\}$ and $r_2 \leqslant \min\{m,q\}$. Solve

$$\min_{\substack{D_2\in\mathbb{R}^{m\times r_2}\\ C_2\in\mathbb{R}^{r_2\times q}}} \min_{\substack{D_1\in\mathbb{R}^{m\times r_1}\\ C_1\in\mathbb{R}^{r_1\times n}}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}_1(\mathbf{y})\|_2^2].$$

$\mathcal{T}_1$: Some results.

- The MSE ε of $\mathcal{T}_1$ is better than BT and GKLT if

$$\sum_{i=r_1+1}^{r} \lambda_i(E_{xy}E_{yy}^{\dagger}E_{yx}) < \sum_{i=1}^{r_2} \lambda_i(E_{xz}E_{zz}^{\dagger}E_{zx})$$

where λ_i is the i-th eigenvalue and $\mathbf{z} = \mathcal{Q}(\mathbf{v}, \mathbf{y})$.

$\mathcal{T}_1$: Some results.

- The MSE ε of $\mathcal{T}_1$ is better than BT and GKLT if

$$\underbrace{\sum_{i=r_1+1}^{r} \lambda_i(E_{xy}E_{yy}^{\dagger}E_{yx})}_{\alpha_y} < \underbrace{\sum_{i=1}^{r_2} \lambda_i(E_{xz}E_{zz}^{\dagger}E_{zx})}_{\alpha_z}$$

where λ_i is the i-th eigenvalue.

$\mathcal{T}_1$: Some results.

Example 1

Consider $\mathbf{x} \in L^2(\Omega, \mathbb{R}^{20})$ and $\mathbf{y} = \mathbf{x} + \xi$, where ξ is a white noise uncorrelated with x and $E_{\xi\xi} = \sigma^2 I_{20}$, where $\sigma = 1$. Let's define an arbitrary random vector $\mathbf{v} \in L^2(\Omega, \mathbb{R}^{20})$, such that $E_{xv} \neq \mathbf{0}$.

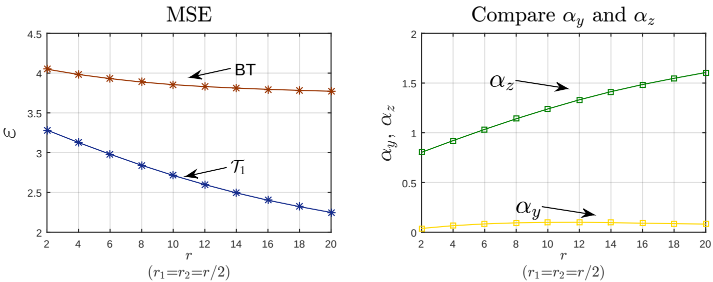


Figure: Average of 10 000 simulations.

$\mathcal{T}_1$: Some results.

- For any random vectors $\mathbf{x}, \mathbf{y}$ and $\mathbf{v}$

$$\mathbb{E}[\|\mathbf{x} - \mathcal{T}_1(\mathbf{y})\|_2^2] = \mathbb{E}[\|\mathbf{x} - D_1 C_1 \mathbf{y}\|_2^2] + \mathbb{E}[\|\mathbf{x} - D_2 C_2 \mathcal{Q}(\mathbf{v}, \mathbf{y})\|_2^2] - \mathbb{E}[\|\mathbf{x}\|_2^2].$$

Therefore, $D_1 C_1$ and $D_2 C_2$ can be computed independently (A computational advantage).

$\mathcal{T}_1$: Some results.

- For any random vectors $\mathbf{x}, \mathbf{y}$ and $\mathbf{v}$

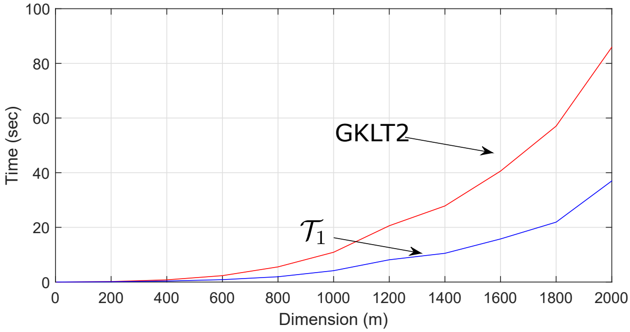
$$\mathbb{E}[\|\mathbf{x} - \mathcal{T}_1(\mathbf{y})\|_2^2] = \mathbb{E}[\|\mathbf{x} - D_1 C_1 \mathbf{y}\|_2^2] + \mathbb{E}[\|\mathbf{x} - D_2 C_2 \mathcal{Q}(\mathbf{v}, \mathbf{y})\|_2^2] - \mathbb{E}[\|\mathbf{x}\|_2^2].$$

Therefore, D_1C_1 and D_2C_2 can be computed independently (A computational advantage).

$\mathcal{T}_1$: Some results.

Example 2

Consider a two Gaussian vectors $\mathbf{x}, \mathbf{v} \in L^2(\Omega, \mathbb{R}^m)$ and a observation $\mathbf{y} = \mathbf{x} + \xi$, where ξ is other Gaussian vector. To estimate covariance matrices, we use training vectors represented by $X, Y, V \in \mathbb{R}^{m \times 3m}$.



$\mathcal{T}_1$: Some results.

- If D_1C_1 and D_2C_2 are full rank matrices, i.e., $\mathcal{T}_1$ is a filter, then the accuracy of $\mathcal{T}_1$ is better than or equal to BT ($r_1 = \min\{m, n\}$ and $r_2 = \min\{m, q\}$).
- The optimal D_2, C_2 and $\mathbf{v}$ that solve the problem satisfy

$$\begin{cases} D_2C_2 = U_{r_2}U_{r_2}(E_{xv} - E_{xy}E_{yy}^{\dagger}E_{yv})(E_{vv} - E_{vy}E_{yy}^{\dagger}E_{yv})^{\dagger} \\ \mathbf{v} = E_{vy}E_{yy}^{\dagger} + (D_2C_2)^{\dagger}\mathbf{x} \end{cases}$$

Remark: Using samples of vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{v}$, it is possible to obtain an estimation of optimal $\mathbf{v}$ and D_2, C_2, through an iterative quadratic minimum distance method.

$\mathcal{T}_1$: Some results.

- If D_1C_1 and D_2C_2 are full rank matrices, i.e., $\mathcal{T}_1$ is a filter, then the accuracy of $\mathcal{T}_1$ is better than or equal to BT ($r_1 = \min\{m, n\}$ and $r_2 = \min\{m, q\}$).
- The optimal D_2, C_2 and $\mathbf{v}$ that solve the problem satisfy

$$\begin{cases} D_2C_2 = U_{r_2}U_{r_2}(E_{xv} - E_{xy}E_{yy}^{\dagger}E_{yv})(E_{vv} - E_{vy}E_{yy}^{\dagger}E_{yv})^{\dagger} \\ \mathbf{v} = E_{vy}E_{yy}^{\dagger} + (D_2C_2)^{\dagger}\mathbf{x} \end{cases}$$

Remark: Using samples of vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{v}$, it is possible to obtain an estimation of optimal $\mathbf{v}$ and D_2, C_2, through an iterative quadratic minimum distance method.

$\mathcal{T}_1$: Some results.

- If D_1C_1 and D_2C_2 are full rank matrices, i.e., $\mathcal{T}_1$ is a filter, then the accuracy of $\mathcal{T}_1$ is better than or equal to BT ($r_1 = \min\{m,n\}$ and $r_2 = \min\{m,q\}$).
- The optimal D_2, C_2 and $\mathbf{v}$ that solve the problem satisfy

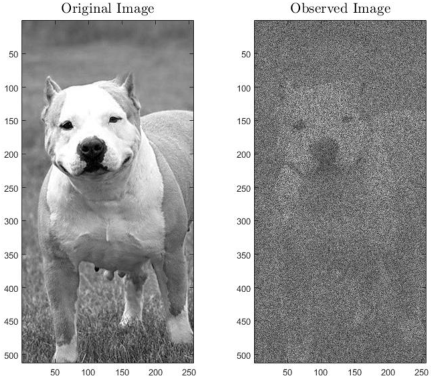
$$\begin{cases} D_2C_2 = U_{r_2}U_{r_2}(E_{xv} - E_{xy}E_{yy}^{\dagger}E_{yv})(E_{vv} - E_{vy}E_{yy}^{\dagger}E_{yv})^{\dagger} \\ \mathbf{v} = E_{vy}E_{yy}^{\dagger} + (D_2C_2)^{\dagger}\mathbf{x} \end{cases}$$

Remark: Using samples of vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{v}$, it is possible to obtain an estimation of optimal $\mathbf{v}$ and D_2, C_2, through an iterative quadratic minimum distance method.

$\mathcal{T}_1$: Some results.

Example 3

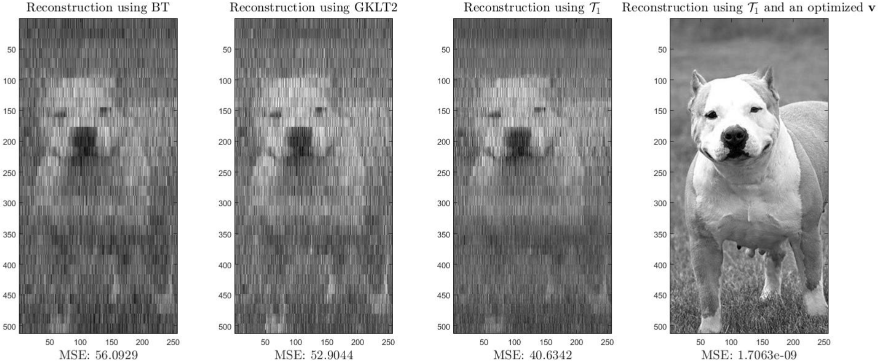
Define: image $X \in \mathbb{R}^{512\times 256}$, observed data $Y \in \mathbb{R}^{512\times 256}$ and arbitrary $V \in \mathbb{R}^{512\times 256}$.



$\mathcal{T}_1$: Some results.

Example 3

Filtering of observed data using BT, GKLT2 and $\mathcal{T}_1$ ($r = 512$).



$\mathcal{T}_1$: Some results.

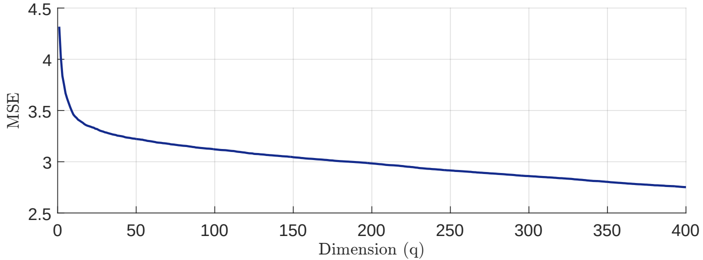
- The MSE of $\mathcal{T}_1$ decreases as the dimension q of vector $\mathbf{v}$ increase.

$\mathcal{T}_1$: Some results.

Example 4

Consider $\mathbf{x} \in L^2(\Omega, \mathbb{R}^{40})$ and $\mathbf{y} = A\mathbf{x} + \xi$, where ξ is a white noise $E_{\xi\xi} = \sigma^2 I_4$, where $\sigma = 1.5$. Let's define an arbitrary random vector $\mathbf{v} \in L^2(\Omega, \mathbb{R}^q)$, such that $E_{xv} \neq \mathbf{0}$.

- If $r = 20$, then MSE of BT is $\varepsilon_{\text{BT}} = 4.9009$.
- If $r_1 = 10$ and $r_2 = 10$ and changing the length q of vector $\mathbf{v}$, then MSE of $\mathcal{T}_1$ is represented in the following plot:

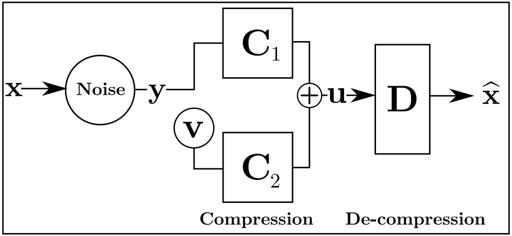


Transform Proposed 2 ($\mathcal{T}_2$)

Transform $\mathcal{T}_2$

$$\mathcal{T}_2(\mathbf{y}) = DC_1\mathbf{y} + DC_2\mathbf{v}$$

- $\mathbf{v} \in L^2(\Omega, \mathbb{R}^q)$ is arbitrary



Problem with $\mathcal{T}_2$

Solve

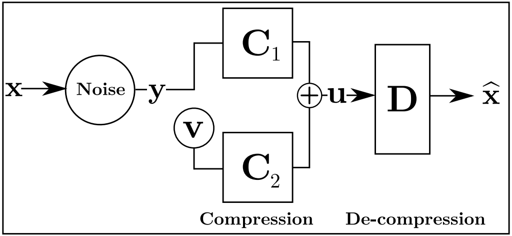
$$\min_{\substack{D\in\mathbb{R}^{m\times r}\\ C_1\in\mathbb{R}^{r\times n}, C_2\in\mathbb{R}^{r\times q}}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}_2(\mathbf{y})\|_2^2].$$

Transform Proposed 2 ($\mathcal{T}_2$)

Transform $\mathcal{T}_2$

$$\mathcal{T}_2(\mathbf{y}) = DC_1\mathbf{y} + DC_2\mathbf{v}$$

- $\mathbf{v} \in L^2(\Omega, \mathbb{R}^q)$ is arbitrary



Problem with $\mathcal{T}_2$

Solve

$$\min_{\substack{D\in\mathbb{R}^{m\times r}\\ C_1\in\mathbb{R}^{r\times n}, C_2\in\mathbb{R}^{r\times q}}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}_2(\mathbf{y})\|_2^2].$$

$\mathcal{T}_2$: Some results.

Let's define the random vector $\mathbf{w} = \begin{bmatrix}\mathbf{y}\\ \mathbf{v}\end{bmatrix}$, and covariance matrix E_{ww}. We consider two scenarios:

1. **S1:** E_{ww} is singular, but $SS^{\dagger}E_{vy} = E_{vy}$, where $S = E_{vv} - E_{vy}E_{yy}^{\dagger}E_{yv}$.
2. **S2:** E_{ww} is nonsingular.

- If **S1** or **S2** are true, then the accuracy of $\mathcal{T}_2$ is better than or equal to BT.
  The GKLT2 is a particular case of $\mathcal{T}_2$ $(\mathbf{v} = \mathbf{y^2})$.

$\mathcal{T}_2$: Some results.

Let's define the random vector $\mathbf{w} = \left[\begin{smallmatrix}\mathbf{y}\\ \mathbf{v}\end{smallmatrix}\right]$, and covariance matrix E_{ww}. We consider two scenarios:

1. **S1:** E_{ww} is singular, but $SS^{\dagger}E_{vy} = E_{vy}$, where $S = E_{vv} - E_{vy}E_{yy}^{\dagger}E_{yv}$.
2. **S2:** E_{ww} is nonsingular.

- If **S1** or **S2** are true, then the accuracy of $\mathcal{T}_2$ is better than or equal to BT.
- The GKLT2 is a particular case of $\mathcal{T}_2$ $(\mathbf{v} = \mathbf{y^2})$.

$\mathcal{T}_2$: Some results.

Let's define the random vector $\mathbf{w} = \left[\begin{smallmatrix}\mathbf{y}\\ \mathbf{v}\end{smallmatrix}\right]$, and covariance matrix E_{ww}. We consider two scenarios:

1. **S1:** E_{ww} is singular, but $SS^{\dagger}E_{vy} = E_{vy}$, where $S = E_{vv} - E_{vy}E_{yy}^{\dagger}E_{yv}$.
2. **S2:** E_{ww} is nonsingular.

- If **S1** or **S2** are true, then the accuracy of $\mathcal{T}_2$ is better than or equal to BT.
  The GKLT2 is a particular case of $\mathcal{T}_2$ ($\mathbf{v} = \mathbf{y}^{\mathbf{2}}$).

$\mathcal{T}_2$: Some results.

Example 5

Consider $\mathbf{x} \in L^2(\Omega, \mathbb{R}^{200})$ and $\mathbf{y} = \mathbf{x} + \xi$, where ξ is a white noise and $E_{\xi\xi} = \sigma^2 I_{200}$, where $\sigma = 0.5$. Let's define an arbitrary random vector $\mathbf{v} \in L^2(\Omega, \mathbb{R}^{200})$, where $E_{xv} \neq \mathbf{0}$.

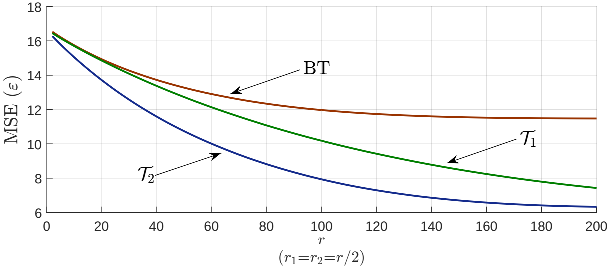


Figure: Average of 10 000 simulations.

$\mathcal{T}_2$: Some results.

- The optimal D, C_1, C_2 and $\mathbf{v}$ that solve the problem satisfy

$$\begin{cases} DC_1 + DC_2 = U_{r_2} U_{r_2} [E_{xy}\ E_{xv}] \begin{bmatrix} E_{yy} & E_{yv} \\ E_{vy} & E_{vv} \end{bmatrix}^{\dagger} \\ \\ \mathbf{v} = (DC_2)^{\dagger}(\mathbf{x} - DC_1\mathbf{y}) \end{cases}$$

Remark: Using samples of vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{v}$, it is possible to obtain an estimation of optimal $\mathbf{v}$ and D, C_1 and C_2, through an iterative quadratic minimum distance method.

$\mathcal{T}_2$: Some results.

- The optimal D, C_1, C_2 and $\mathbf{v}$ that solve the problem satisfy

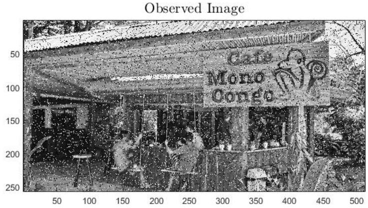
$$\begin{cases} DC_1 + DC_2 = U_{r_2} U_{r_2} [E_{xy} \; E_{xv}] \begin{bmatrix} E_{yy} & E_{yv} \\ E_{vy} & E_{vv} \end{bmatrix}^{\dagger} \\ \\ \mathbf{v} = (DC_2)^{\dagger} (\mathbf{x} - DC_1 \mathbf{y}) \end{cases}$$

Remark: Using samples of vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{v}$, it is possible to obtain an estimation of optimal $\mathbf{v}$ and D, C_1 and C_2, through an iterative quadratic minimum distance method.

$\mathcal{T}_2$: Some results.

Example 6

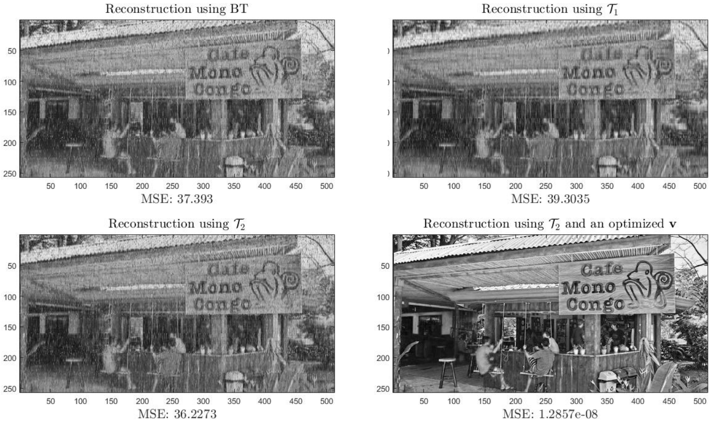
Define: image $X \in \mathbb{R}^{256\times512}$, observed data $Y \in \mathbb{R}^{256\times512}$ and arbitrary $V \in \mathbb{R}^{256\times512}$.



$\mathcal{T}_2$: Some results.

Example 6

Reconstruction of observed data using BT, $\mathcal{T}_1$and $\mathcal{T}_2$ as when $r = 128$.



$\mathcal{T}_2$: Some results.

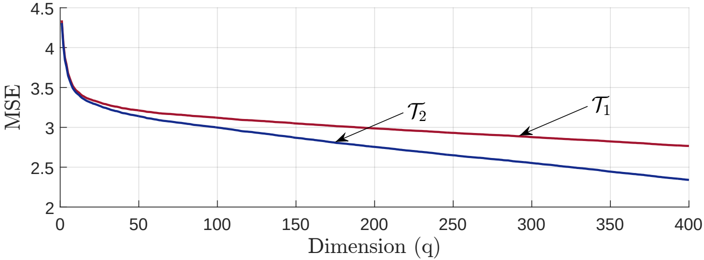
- The error associated ε decreases as the dimension q of vector $\mathbf{v}$ increase.

$\mathcal{T}_2$: Some results.

Example 7

Consider $\mathbf{x} \in L^2(\Omega, \mathbb{R}^{40})$ and $\mathbf{y} = A\mathbf{x} + \xi$, where ξ is a white noise $E_{\xi\xi} = \sigma^2 I_4$, where $\sigma = 1.5$. Let's define an arbitrary random vector $\mathbf{v} \in L^2(\Omega, \mathbb{R}^q)$, such that $E_{xv} \neq \mathbf{0}$.

- If $r = 20$, then MSE of BT is $\varepsilon_{\text{BT}} = 4.9009$.
- If $r_1 = 10$, $r_2 = 10$, $r = 20$ and changing the length q of vector $\mathbf{v}$, then MSE of $\mathcal{T}_1$ and $\mathcal{T}_2$ are represented in the following plot:

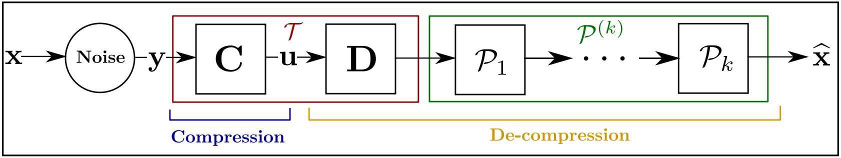


Transform Proposed 3 ($\mathcal{T}_3^{(k)}$)

Transform $\mathcal{T}_3^{(k)}$

$$\mathcal{T}_3^{(k)}(\mathbf{y}) = (\mathcal{P}^{(k)} \circ \mathcal{T})(\mathbf{y})$$

- $\mathcal{T}$ is the BT or $\mathcal{T}_2$.
- $\mathcal{P}^{(k)} = \mathcal{P}_k \circ \cdots \circ \mathcal{P}_1$.
- Each $\mathcal{P}_i$ is filter.



Problem with $\mathcal{T}_3^{(k)}$

Solve

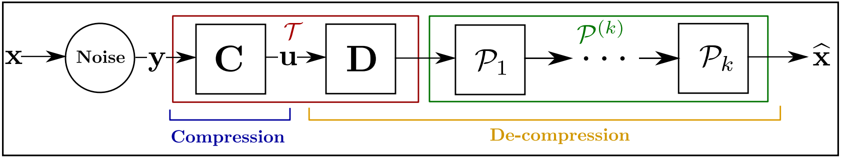
$$\min_{\mathcal{T}, \mathcal{P}_1, \ldots, \mathcal{P}_k} \mathbb{E}[\|\mathbf{x} - \mathcal{T}_3^{(k)}(\mathbf{y})\|_2^2].$$

Transform Proposed 3 ($\mathcal{T}_3^{(k)}$)

Transform $\mathcal{T}_3^{(k)}$

$$\mathcal{T}_3^{(k)}(\mathbf{y}) = (\mathcal{P}^{(k)} \circ \mathcal{T})(\mathbf{y})$$

- $\mathcal{T}$ is the BT or $\mathcal{T}_2$.
- $\mathcal{P}^{(k)} = \mathcal{P}_k \circ \cdots \circ \mathcal{P}_1$.
- Each $\mathcal{P}_i$ is filter.



Problem with $\mathcal{T}_3^{(k)}$

Solve

$$\min_{\mathcal{T}, \mathcal{P}_1, \ldots, \mathcal{P}_k} \mathbb{E}[\|\mathbf{x} - \mathcal{T}_3^{(k)}(\mathbf{y})\|_2^2].$$

$\mathcal{T}_3^{(k)}$: Some results.

Transform $\mathcal{T}_3^{(k)}$

$$\begin{aligned} \mathcal{T}_3^{(k)}(\mathbf{y}) &= (\mathcal{P}^{(k)} \circ \mathcal{T})(\mathbf{y}) \\ &= (\mathcal{P}_k \circ \cdots \circ \mathcal{P}_1 \circ \mathcal{T})(\mathbf{y}) \end{aligned}$$

- Each $\mathcal{P}_i(\mathbf{y})$ **need to be** a second degree transform, i.e.,

$$\mathcal{P}_i(\mathbf{y}) = F_{1,i}\mathbf{y} + F_{2,i}\mathbf{v}_i,$$

because, if each $\mathcal{P}_i$ is a linear transform, i.e.,

$$\mathcal{P}_i(\mathbf{y}) = F_i\mathbf{y},$$

then **the accuracy of $\mathcal{T}_3^{(k)}$ never will be better than $\mathcal{T}$.**

$\mathcal{T}_3^{(k)}$: Some results.

Scheme of solution of $\mathcal{T}_3^{(k)}(\mathbf{y}) = (\mathcal{P}_k \circ \cdots \circ \mathcal{P}_1 \circ \mathcal{T})(\mathbf{y})$

Step 1: Compute compressor and de-compresor from the problem

$$\min_{\mathcal{T}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}(\mathbf{y})\|_2^2].$$

Step 2: Define $\mathbf{t}_0 = \mathcal{T}(\mathbf{y})$ and compute $\mathcal{P}_1$ from

$$\min_{\mathcal{P}_1} \mathbb{E}[\|\mathbf{x} - \mathcal{P}_1(\mathbf{t}_1)\|_2^2].$$

Step 3: Define $\mathbf{t}_i = (\mathcal{P}_i \circ \cdots \circ \mathcal{P}_1 \circ \mathcal{T})(\mathbf{y})$ and compute $\mathcal{P}_{i+1}$ from

$$\min_{\mathcal{P}_{i+1}} \mathbb{E}[\|\mathbf{x} - \mathcal{P}_{i+1}(\mathbf{t}_i)\|_2^2],$$

for $i = 1, ..., k-1$.

$\mathcal{T}_3^{(k)}$: Some results.

Scheme of solution of $\mathcal{T}_3^{(k)}(\mathbf{y}) = (\mathcal{P}_k \circ \cdots \circ \mathcal{P}_1 \circ \mathcal{T})(\mathbf{y})$

Step 1: Compute compressor and de-compresor from the problem

$$\min_{\mathcal{T}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}(\mathbf{y})\|_2^2].$$

Step 2: Define $\mathbf{t}_0 = \mathcal{T}(\mathbf{y})$ and compute $\mathcal{P}_1$ from

$$\min_{\mathcal{P}_1} \mathbb{E}[\|\mathbf{x} - \mathcal{P}_1(\mathbf{t}_1)\|_2^2].$$

Step 3: Define $\mathbf{t}_i = (\mathcal{P}_i \circ \cdots \circ \mathcal{P}_1 \circ \mathcal{T})(\mathbf{y})$ and compute $\mathcal{P}_{i+1}$ from

$$\min_{\mathcal{P}_{i+1}} \mathbb{E}[\|\mathbf{x} - \mathcal{P}_{i+1}(\mathbf{t}_i)\|_2^2],$$

for $i = 1, ..., k-1$.

$\mathcal{T}_3^{(k)}$: Some results.

Scheme of solution of $\mathcal{T}_3^{(k)}(\mathbf{y}) = (\mathcal{P}_k \circ \cdots \circ \mathcal{P}_1 \circ \mathcal{T})(\mathbf{y})$

Step 1: Compute compressor and de-compresor from the problem

$$\min_{\mathcal{T}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}(\mathbf{y})\|_2^2].$$

Step 2: Define $\mathbf{t}_0 = \mathcal{T}(\mathbf{y})$ and compute $\mathcal{P}_1$ from

$$\min_{\mathcal{P}_1} \mathbb{E}[\|\mathbf{x} - \mathcal{P}_1(\mathbf{t}_1)\|_2^2].$$

Step 3: Define $\mathbf{t}_i = (\mathcal{P}_i \circ \cdots \circ \mathcal{P}_1 \circ \mathcal{T})(\mathbf{y})$ and compute $\mathcal{P}_{i+1}$ from

$$\min_{\mathcal{P}_{i+1}} \mathbb{E}[\|\mathbf{x} - \mathcal{P}_{i+1}(\mathbf{t}_i)\|_2^2],$$

for $i = 1, ..., k-1$.

$\mathcal{T}_3^{(k)}$: Some results.

- The MSE of transform $\mathcal{T}_3^{(k)}$ is less than transform $\mathcal{T}$, for all $k \geqslant 1$.
- If $k > j$, then the accuracy of transform $\mathcal{T}_3^{(k)}$ is better than transform $\mathcal{T}_3^{(j)}$.

$\mathcal{T}_3^{(k)}$: Some results.

- The MSE of transform $\mathcal{T}_3^{(k)}$ is less than transform $\mathcal{T}$, for all $k \geqslant 1$.
- If $k > j$, then the accuracy of transform $\mathcal{T}_3^{(k)}$ is better than transform $\mathcal{T}_3^{(j)}$.

$\mathcal{T}_3^{(k)}$: Some results.

Example 8 (Face reconstruction)

We consider a training set $\{\mathbf{x}^{(k)}, \mathbf{y}^{(k)}\}_{k=0}^{1520}$ consisted of 1521 faces images.

- $\mathbf{x}^{(k)} \in \mathbb{R}^{109824}$ were obtained using **BioID Face Database** [Bio] consists of a set $\{X^{(k)}\}_{k=1}^{1520}$ of grayscale images of 23 different persons, $X^{(k)} \in \mathbb{R}^{286\times 384}$, where

$$\mathbf{x}^{(k)} = \texttt{vec}(X^{(k)}).$$

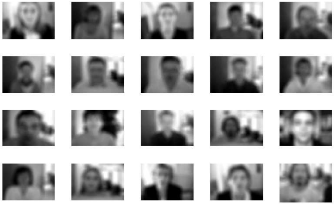
- To generate the corresponding noisy $\mathbf{y}^{(k)} \in \mathbb{R}^{109824}$ (blurred image), we use MATLAB command `imfilter` to create a blurred image $Y^{(k)}$. Finally,

$$\mathbf{y}^{(k)} = \texttt{vec}(Y^{(k)}).$$

$\mathcal{T}_3^{(k)}$: Some results.

Example 8 (Face reconstruction)

(a) Source images.

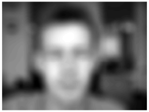


(b) Blurred images.

$\mathcal{T}_3^{(k)}$: Some results.

Example 8 (Face reconstruction)

Reconstruction of $\widetilde{Y}$, using $r = 30$, $\mathcal{T} = \mathcal{T}_2$ and $\mathcal{P}_i = F_{1,i}\mathbf{y} + F_{2,i}\mathbf{y}^2$.



Observed image $\widetilde{Y}$

Source image

BT
MSE$= 1.127 \times 10^2$

GKLT2/$\mathcal{T}_1$
MSE$= 9.812 \times 10^1$

$\mathcal{T}_3^{(1)}$
MSE$= 1.75 \times 10^1$

$\mathcal{T}_3^{(2)}$
MSE$= 6.65 \times 10^{-10}$

Conclusions

- We propose new transforms to solve the problem

$$\min_{\mathcal{T}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}(\mathbf{y})\|_2^2].$$

- The new transforms allow **compression**, **de-compression** and **filtering** of vector $\mathbf{y}$.

- The proposed transforms improve the accuracy of known methods, i.e., BT, GKLT and GKLT2.

Conclusions

- We propose new transforms to solve the problem

$$\min_{\mathcal{T}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}(\mathbf{y})\|_2^2].$$

- The new transforms allow **compression**, **de-compression** and **filtering** of vector $\mathbf{y}$.

- The proposed transforms improve the accuracy of known methods, i.e., BT, GKLT and GKLT2.

Conclusions

- We propose new transforms to solve the problem

$$\min_{\mathcal{T}} \mathbb{E}[\|\mathbf{x} - \mathcal{T}(\mathbf{y})\|_2^2].$$

- The new transforms allow **compression**, **de-compression** and **filtering** of vector $\mathbf{y}$.

- The proposed transforms improve the accuracy of known methods, i.e., BT, GKLT and GKLT2.

Conclusions

- The improvement in accuracy is achieved by special structures of the proposed transforms which contain more parameters to optimize compared to the known transforms.

Table: **Matrices and parameters to optimize for each transform.**

Transforms	Matrices to optimize	Number of parameters to optimize
BT	D, C	$r(m+n)$
GKLT	F	mn
GKLT2	F_1, F_2	$2mn$
$\mathcal{T}_1$	D_1, C_1, D_2, C_2	$r_1(m+n) + r_2(m+q)$
$\mathcal{T}_2$	D, C_1, C_2	$r(m+n+q)$
$\mathcal{T}_3^{(k)}$ (*)	$D, C,$ $F_{1,1}, F_{2,1}, ..., F_{1,k}, F_{2,k}$	$r(m+n) + m(mk + \sum_{i=1}^k q_i)$
$\mathcal{T}_3^{(k)}$ (**)	$D, C_1, C_2,$ $F_{1,1}, F_{2,1}, ..., F_{1,k}, F_{2,k}$	$r(m+n+q) + m(mk + \sum_{i=1}^k q_i)$

(*) $\mathcal{T} = \text{BT}$ (**) $\mathcal{T} = \mathcal{T}_2$.

Conclusions

- The improvement in accuracy is achieved by special structures of the proposed transforms which contain more parameters to optimize compared to the known transforms.

Table: **Matrices and parameters to optimize for each transform.**

Transforms	**Matrices to optimize**	**Number of parameters to optimize**
BT	D, C	$r(m+n)$
GKLT	F	mn
GKLT2	F_1, F_2	$2mn$
$\mathcal{T}_1$	D_1, C_1, D_2, C_2	$r_1(m+n) + r_2(m+q)$
$\mathcal{T}_2$	D, C_1, C_2	$r(m+n+q)$
$\mathcal{T}_3^{(k)}$ (*)	$D, C,$ $F_{1,1}, F_{2,1}, ..., F_{1,k}, F_{2,k}$	$r(m+n) + m(mk + \sum_{i=1}^k q_i)$
$\mathcal{T}_3^{(k)}$ (**)	$D, C_1, C_2,$ $F_{1,1}, F_{2,1}, ..., F_{1,k}, F_{2,k}$	$r(m+n+q) + m(mk + \sum_{i=1}^k q_i)$

(*) $\mathcal{T}$ = BT (**) $\mathcal{T} = \mathcal{T}_2$.

Future Work

- Develop a method to find an optimal random vector $\mathbf{v}$ in $\mathcal{T}_1$ and in $\mathcal{T}_2$:

$$\min_{\mathbf{v}\in L^2(\Omega,\mathbb{R}^q)} \min_{\substack{D_2\in\mathbb{R}^{m\times r_2}\\ C_2\in\mathbb{R}^{r_2\times q}}} \min_{\substack{D_1\in\mathbb{R}^{m\times r_1}\\ C_1\in\mathbb{R}^{r_1\times n}}} \mathbb{E}[\|\mathbf{x}-\mathcal{T}_1(\mathbf{y})\|_2^2].$$

$$\min_{\mathbf{v}\in L^2(\Omega,\mathbb{R}^q)} \min_{\substack{D\in\mathbb{R}^{m\times r}\\ C_1\in\mathbb{R}^{r\times n}, C_2\in\mathbb{R}^{r\times q}}} \mathbb{E}[\|\mathbf{x}-\mathcal{T}_2(\mathbf{y})\|_2^2].$$

- In transforms $\mathcal{T}_1$ and $\mathcal{T}_2$, what happen when dimension q of vector $\mathbf{v}$ takes to infinity?

Future Work

- Develop a method to find an optimal random vector $\mathbf{v}$ in $\mathcal{T}_1$ and in $\mathcal{T}_2$:

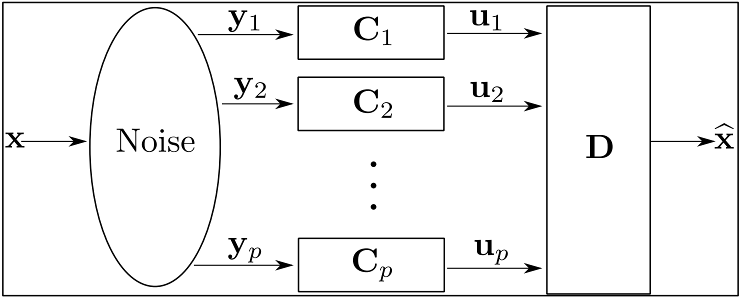
$$\min_{\mathbf{v}\in L^2(\Omega,\mathbb{R}^q)} \min_{\substack{D_2\in\mathbb{R}^{m\times r_2}\\ C_2\in\mathbb{R}^{r_2\times q}}} \min_{\substack{D_1\in\mathbb{R}^{m\times r_1}\\ C_1\in\mathbb{R}^{r_1\times n}}} \mathbb{E}[\|\mathbf{x}-\mathcal{T}_1(\mathbf{y})\|_2^2].$$

$$\min_{\mathbf{v}\in L^2(\Omega,\mathbb{R}^q)} \min_{\substack{D\in\mathbb{R}^{m\times r}\\ C_1\in\mathbb{R}^{r\times n}, C_2\in\mathbb{R}^{r\times q}}} \mathbb{E}[\|\mathbf{x}-\mathcal{T}_2(\mathbf{y})\|_2^2].$$

- In transforms $\mathcal{T}_1$ and $\mathcal{T}_2$, what happen when dimension q of vector $\mathbf{v}$ takes to infinity?

Future Work

- Use the proposed transforms in a distributed system scenario.



Publications

- A. Torokhti and P. Soto-Quiros, "Generalized Brillinger-Like Transforms," in *IEEE Signal Processing Letters*, vol. 23, no. 6, pp. 843-847, June 2016.
- A. Torokthi, S. Miklavcic and P. Soto-Quiros, "Distributed Systems: Identification, Optimization and Simulations", in *International Journal of Electronics and Electrical Engineering*, vol. 4, no. 4, pp. 322-327, 2016.
- A. Torokhti and P. Soto-Quiros, "Optimal Transforms of Random Vectors: the Case of Successive Optimizations" (Submitted)

References I

[Bio] BioID-Company.

Bioid face database.

[Bri75] D. Brillinger.

Time Series: Data Analysis and Theory.

Holt Rinehart, 1975.

[HL98] Y. Hua and Wanquan Liu.

Generalized Karhunen-Loeve transform.

IEEE Signal Processing Letters, 5(6):141–142, June 1998.

[TH01] A. Torokhti and P. Howlett.

Optimal fixed rank transform of the second degree.

IEEE Trans. CAS. Part II, Analog and Digital Signal Processing, 48(3):309 – 315, 2001.

References II

[TH07] A. Torokhti and P. Howlett.
Computational Methods for Modelling of Nonlinear Systems.
Elsevier, 2007.